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а.Нешитов

$U \subset V$, $G \curvearrowright U$ свободно,

$\operatorname{codim} V\setminus U \geq 3$, V — лин. предст. G (G — расщ. полупростая)

$$\begin{tikzcd}
Y_\zeta \arrow[r] \arrow[d] & U \arrow[d] & \\
X_\zeta \arrow[r] \arrow[d] & U/T \arrow[d] & T \arrow[l] \arrow[d] \\
\zeta \arrow[r] & U/G & Z \arrow[l]
\end{tikzcd}$$

$\zeta = \operatorname{Spec} F(U/G)$

$X_\zeta = Y_\zeta/T$, $Z \in (U/G)^{(1)}$

$$\begin{tikzcd}
0 \arrow[r] & A^1(X_\zeta, K_2) \arrow[r] \arrow[d, "\partial^\times"] & F(U/G)^\times \otimes \Lambda \arrow[r, "p"] \arrow[d, "\partial"] & \widetilde{H}^3(F(U/G),2) \arrow[r, "\alpha"] \arrow[d, "\partial"] & CH^2(X_\zeta)_{tors} \\
0 \arrow[r] & \bigoplus A^1(X_\zeta, K_1) \arrow[r] & \bigoplus\limits_{Z\in U/G^{(1)}} \Lambda \arrow[r] & \bigoplus\limits_{Z\in (U/G)^{(1)}} \widetilde{H}^2(F(Z),1) \arrow[r] & 0
\end{tikzcd}$$

$\bigoplus A^1(X_\zeta, K_1) = \operatorname{Pic} X_Z$

$$\widetilde{H}^3(F(U/G),2) = \operatorname{Ker}\left(H^3(F(U/G),2) \to H^3(F(X_\zeta),2)\right)$$

$$0 \to \operatorname{im} p \cap \operatorname{Ker}\partial \to \operatorname{Ker}\partial \to CH^2(X_\zeta)_{tors} \to 0$$

$\operatorname{Ker}\partial = \operatorname{Inv}^3(G,2)_{norm}$

$$0 \to p(F^\times \otimes \Lambda) \to \operatorname{im} p \cap \operatorname{Ker}\partial \to \operatorname{coker}\partial^\times \to 0$$

$p(F^\times \otimes \Lambda) = \operatorname{Inv}^3(G,2)_{dec}$

$$0 \to \operatorname{coker}\partial^{\times} \to \mathrm{Inv}^3(G,2)_{ind} \to CH^2(X_\zeta)_{tors} \to 0$$

$$CH^2(U/G) \to CH^2(U/T) \to CH^2(X_\zeta) \to 0$$

$$\begin{tikzcd}
\mathbb{Z}[T^*] \arrow[r, equal] & K_0^T(F) \arrow[r] & K_0^T(U) = K_0(U/T) \arrow[rr, "c_2"] & & CH^2(U/T) \\
\mathbb{Z}[T^*]^W \arrow[u] \arrow[r, equal] & K_0^G(F) \arrow[u] \arrow[r] & K_0^G(U) \arrow[u] \arrow[r] & K_0(U/G) \arrow[r] & CH^2(U/G) \arrow[u]
\end{tikzcd}$$

$$c_2 : \tau^2(U/T)/\tau^3(U/T) \simeq CH^2(U/T)$$

$\rightsquigarrow$ $\operatorname{im}(CH^2(U/G) \to CH^2(U/T)) = c_2(\mathbb{Z}[T^*]^W)$

Более того, $CH^2(U/T) \cong S^2(T^*)$ //см. ниже

$$\rightsquigarrow c_\bullet : \mathbb{Z}[T^*] \to S^\bullet(T^*)[[t]]^\times$$
$$e^\lambda \mapsto 1 - \lambda t$$

c_2 – второй член в разложении по t:

$$c_2\Big(\sum_i e^{\lambda_i}\Big) = \sum_{i<j} \lambda_i \lambda_j$$

$$\begin{tikzcd}
\tau^3(U/T) \arrow[r] \arrow[d] & \tau^2(U/T) \arrow[r] \arrow[d] & CH^2(U/T) \arrow[r] \arrow[d] & 0 \\
\tau^3(X_\zeta) \arrow[r] \arrow[d, hook] & \tau^2(X_\zeta) \arrow[r] \arrow[d, hook] & CH^2(X_\zeta) \arrow[r] \arrow[d] & 0 \\
\tau^3(X_\zeta^{sep}) \arrow[r] & \tau^2(X_\zeta^{sep}) \arrow[r] & CH^2(X_\zeta^{sep}) \arrow[r] & 0
\end{tikzcd}$$

$X_\zeta^{sep} := X_\zeta \times F(\zeta)^{sep}$ — расщепимое мн-ие флагов
$\quad \cong G/T \times F(\zeta)^{sep}$

$$\rightsquigarrow \quad c_2(\mathrm{Ker}(K_0(U/T) \to K_0(X_\zeta^{sep}))) = \mathrm{Ker}(CH^2(U/T) \to CH^2(X_\zeta))$$

$$c_2(\mathrm{Ker}(\underset{\substack{\| \\ \mathbb{Z}[T^*]}}{K_0^T(F)} \to K_0(X_\zeta^{sep}) = K_0(G/T)))$$

односвязная

$$\widetilde{G} \longrightarrow G$$

$$G/T = \widetilde{G}/\widetilde{T}$$

$$\begin{tikzcd}
\mathbb{Z}[T^*] \arrow[r] \arrow[d, hook] & K_0(G/T) \arrow[d, equal] \\
\mathbb{Z}[\Lambda] = \mathbb{Z}[\widetilde{T}^*] & K_0(\widetilde{G}/\widetilde{T})
\end{tikzcd}$$

$$\begin{tikzcd}
\mathbb{Z}[T^*] = K_0^T(F) \arrow[r] \arrow[d] & K_0(U/T) \arrow[d] \\
S^2(T^*) \arrow[r, "\sim"] & CH^2(U/T)
\end{tikzcd}$$

$$\lambda_1 \lambda_2 \longmapsto c_1(\mathcal{L}_{\lambda_1}) \cdot c_1(\mathcal{L}_{\lambda_2})$$

т. Шевалле: $\mathrm{Ker}(\mathbb{Z}[\Lambda] \to K_0(\widetilde{G}/\widetilde{T})) = \widetilde{I}^w$

где $\widetilde{I} = \mathrm{Ker}(\mathbb{Z}[\Lambda] \longrightarrow \mathbb{Z})$, $\widetilde{I}^w = \langle x \mid x \in \widetilde{I} \cap \mathbb{Z}[\Lambda]^w \rangle$

$$e^\lambda \longmapsto 1$$

$$\left(K_0(G/T) = R(T) \otimes_{R(G)} \mathbb{Z} \right)$$

$$\rightsquigarrow \mathrm{Ker}(CH^2(U/T) \longrightarrow CH^2(X_\zeta)) = c_2(\widetilde{I}^w \cap \mathbb{Z}[T^*])$$

$$\rightsquigarrow \mathrm{coker}\, \partial^* = c_2(\widetilde{I}^w \cap \mathbb{Z}[T^*]) \Big/ c_2(\mathbb{Z}[T^*]^w)$$

$\mathrm{Inv}^3(G,2)_{ind} = CH^2(X_\zeta)_{tors} = 0$ в след. случаях:

adjoint

- A_n
- B_n
- $C_n,\ n \not\vdots\ 4$
- $D_n,\ n \not\vdots\ 4$

- $C_{4n} \longrightarrow \mathbb{Z}/2$
- $E_6 \longrightarrow \mathbb{Z}/2$
- $E_7 \longrightarrow \mathbb{Z}/3$
- $D_{4n} \longrightarrow \mathbb{Z}/2$