21.03.2006

Sourour, 1986

$a_1,\dots,a_{n-1}\in k$, $a_i\neq 0$

$X\in GL(n,k)$, X — не центральная,

$\leadsto \exists Y : YXY^{-1}=A$: ее посл-сть гл. миноров — строка $(a_1,\dots,a_{n-1})$

На самом деле $a_i\neq 0$ — неважно. (Gordeev)

$$\prod_{i=1}^{n}\beta_i\gamma_i=\det A$$

$\leadsto A=BC$, $B=\begin{pmatrix} * & 0 \\ * & * \end{pmatrix}$, $C=\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}$ — по сути у Sourour

$B_{ii}=\beta_i$ $\quad C_{ii}=\gamma_i$

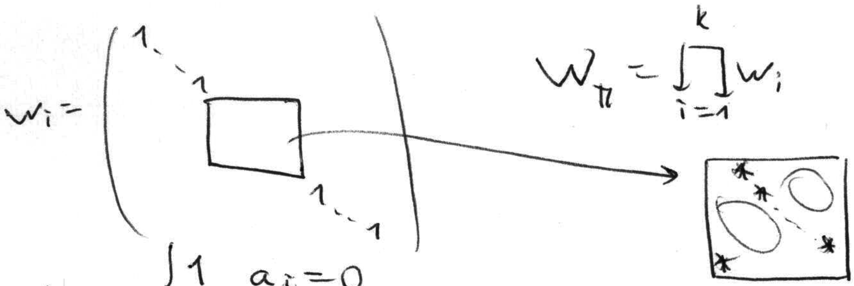
Thm $X\in GL_n(K)$, $n>1$, X — не центрально

$\leadsto \forall (a_1,\dots,a_{n-1})$, $a_i\in k$

$\exists Y\in SL_n(k)$: $\delta_i(YXY^{-1})=a_i \quad i=\overline{1,n-1}$

$\underbrace{00\dots0}_{w_1}\dots\underbrace{0\dots0}_{w_2}\dots\underbrace{0\dots0}_{w_k}$.

$\underbrace{00\dots0x}$



$W_\pi=\prod_{i=1}^{k}w_i$

$H_{ii}=\begin{cases}1, & a_i=0\\ \delta_i(H)=a_i, & a_i\neq 0\end{cases}$

diag

$w_\pi\cdot H$

$\longrightarrow YXY^{-1}=V w_\pi H U$

$V=\begin{pmatrix}1 & 0\\ * & 1\end{pmatrix}$, $U=\begin{pmatrix}1 & *\\ 0 & 1\end{pmatrix}$

доп. условие:

$1\notin\pi \Rightarrow V_{21}\neq 0$

$n=2$, $a_1\neq 0$

$a_1=0$: $Y_1XY_1^{-1}=\begin{pmatrix}1 & 0\\ v & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & *\end{pmatrix}\begin{pmatrix}1 & *\\ 0 & 1\end{pmatrix}$

Переход $n \geq 3$

Разложение Шевалле – Мацумото:

$$YXY^{-1} = \begin{pmatrix} 1 & & & 0 \\ v_1 & 1 & & \\ \vdots & & \ddots & \\ v_n & 0 & & 1 \end{pmatrix} \left(\begin{array}{c|c} h & 0 \\ \hline 0 & * \end{array} \right) \begin{pmatrix} 1 & u_1 & \dots & u_n \\ & 1 & & 0 \\ & & \ddots & \\ & 0 & & 1 \end{pmatrix}$$

$$\forall h,\ h \neq 0$$

(верно $\forall$ max парабол. подгр. в $\forall$ гр. Шевалле)

$\boxed{\text{Случаи}}$ (1) $a_1, a_2 = 0$ (3) $a_1 = 0,\ a_2 \neq 0$

(2) $a_1 \neq 0,\ a_2 = 0$ (4) $a_1 \neq 0,\ a_2 \neq 0$

$\boxed{1}$

$W_\pi = W_{(12)} W'_\pi$

$$Y_1 X Y_1^{-1} = \begin{pmatrix} 1 & & 0 \\ \vdots & \ddots & \\ & 0 & 1 \end{pmatrix} \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & * \end{array} \right) \begin{pmatrix} 1 & & \\ & 1 & 0 \\ & 0 & \ddots \ 1 \end{pmatrix}$$

$$\parallel$$

$$X$$

$$Y_2 = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & * \end{array} \right) \longrightarrow \text{индукционное предположение}$$

$\boxed{2}$ $v_{21} \neq 0 \Rightarrow V = V' \circ t_{21},\ V'_{21} = 0$

$$\exists t_{12}, \tilde{t}_{12} : t_{12} t_{21} = w_{(12)} \tilde{t}_{12}$$

$$t_{12} X t_{12}^{-1} = t_{12} V w'_\pi H U t_{12}^{-1} = t_{12} V' t_{21} w'_\pi H U t_{12}^{-1} =$$

$$= \underbrace{t_{12} V' t_{12}^{-1}}_{\tilde{V}} \underbrace{t_{12} t_{21}}_{w_{(12)} \tilde{t}_{12}} w'_\pi H U t_{12}^{-1} =$$

$$= \tilde{V} (w_{(12)} w'_\pi H) \underbrace{\tilde{t}_{12} \cdot [\tilde{t}_{12}^{-1}, (w'_\pi H)^{-1}]}_{H^{-1} w'_\pi t_{12} w'_\pi H = t_{1k}} \tilde{U}$$

в унип. радикале

$$\parallel$$

$$\tilde{V} (\ldots) \underbrace{t_{1k} \tilde{U}}_{\begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix}}$$

(б) $V_{21} = 0$

$t_{12} X t_{12}^{-1}$ $\quad u_{12} = 0$

$\searrow v_{21} \neq 0$

$\boxed{2}$ $1 \in Mob\,\pi$, $2 \in Mob\,\pi$

$a_1 \neq 0$, $a_2 = 0$

$$w_\pi H = \begin{pmatrix} 1 & 0 \\ 0 & * \end{pmatrix}$$

$$Y X Y^{-1} = V w_\pi H U$$

$\boxed{3}$ $a_1 = 0$, $a_2 \neq 0$

(a) $V_{21} \neq 0 \Longrightarrow$ —"— (1)

$$H^{-1} w_\pi^{-1} \tilde{t}_{12} w_\pi H = \tilde{\tilde{t}}_{12}$$

(б) $V_{21} = 0$, $[w_\pi^{-1} H, t_{12}] \neq 1$

$$H = \begin{pmatrix} 1 & 0 & \\ 0 & * & 0 \\ & 0 & \ddots \end{pmatrix}, \ * \neq 1$$

$\sim Y X Y^{-1} \rightsquigarrow v_{21} \neq 0$

(c) $V_{21} = 0$ $[w_\pi H, t_{12}] = 1$, $u_{12} \neq 0$

$\rightarrow$ сопрягаем w_{12} і $w_{12} X w_{12}^{-1}$

(d) $u_{12} = 0$, $v_{21} = 0$, $[\ ,\ ] = 1$

$\boxed{4}$ $a_1, a_2 \neq 0$