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$(R, \Lambda)$ — форм. кольцо

$\alpha \longmapsto \bar{\alpha}$ — адд., антигомоморфизм

$\Lambda_{min} \subseteq \Lambda \subseteq \Lambda_{max}$

$\Lambda_{min} = \{\alpha - \lambda\bar{\alpha}\}$, $\quad \Lambda_{max} = \{\alpha \mid \alpha = -\lambda\bar{\alpha}\}$

$\alpha \Lambda \bar{\alpha} \subseteq \Lambda$

$U(2n, R, \Lambda) \leq GL(2n, R)$

$e_1 \dots e_n \; e_{-n} \dots e_{-1}$ — базис

$StU(2n, R, \Lambda)$ — образующие $X_{ij}(\xi)$,

$i \neq j$, и а) если $i \neq -j$, то $\xi \in R$

б) а если $i = -j$, то $\xi \in \lambda^{-\left(\frac{\epsilon(i)+1}{2}\right)}\Lambda$

$\epsilon(i) = \mathrm{sign}(i)$

R1) $\cancel{X}_{ij}(\xi) = \cancel{X}_{-j,-i}\left(\lambda^{\frac{\epsilon(j)-\epsilon(i)}{2}} \bar{\xi}\right)$

R2) $X_{ij}(\xi) X_{ij}(\zeta) = X_{ij}(\xi + \zeta)$

R3) $[X_{ij}(\xi), X_{hk}(\zeta)] = e$, если $h \neq j, -i$, $k \neq i, -j$

R4) $[X_{ij}(\xi), X_{jh}(\zeta)] = X_{ih}(\xi\zeta)$, $i, h \neq \pm j$, $i \neq \pm h$

R5) $[X_{ij}(\xi), X_{j,-i}(\zeta)] = X_{i,-i}(\dots)$, $i \neq \pm j$

R6) $[X_{i,-i}(\alpha), X_{-i,j}(\xi)] = X_{i,j}(\alpha\xi) X_{-j,j}\left(-\lambda^{\frac{\epsilon(j)+\epsilon(i)}{2}} \bar{\xi}\alpha\xi\right)$

$StUP_n = \langle X_{ij}(\xi) \mid i > 0 \rangle$

$StUP_n = StUL_n \ltimes StUU_n$

$StUL_n = \langle X_{ij}(\xi) \mid i, j > 0 \rangle$, $\quad StUU_n = \langle X_{ij}(\xi) \mid i > 0, j < 0 \rangle$

$StUP_1 = \langle X_{ij} \mid i \neq -1, j \neq 1 \rangle$

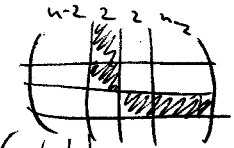
<u>Теорема</u> Если $sr R < n-2$, то

$StU(2n, R, \Lambda) = StUP_n \, StUU_n^- \, StUP_1$

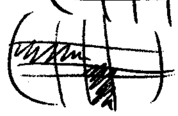
(у Bak–Petrov–Tang — то же $sr R < n-2$

+ $\forall$ унимод. столбца $\exists$ антиэрмитова $a \in AH_{R,\Lambda}(n-1)$

$\begin{pmatrix} u^+ \\ u^- \end{pmatrix} = u \in R^{2(n-1)}$ : $u^+ + au^-$ — унимодулярна

$V^+ =$ 

$\leqslant StUL_n$

$V^- =$ 

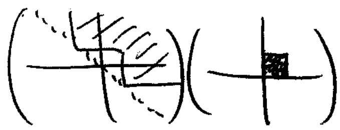
$U^+ = V^+ \ltimes StUU_n^+$

$U^- = V^- \ltimes StUU_n^-$

$Y^+ = X_{n,-n} X_{n-1,-(n-1)} X_{n-1,-n}$

$Y^- = X_{-n,n} X_{-(n-1),n-1} X_{-(n-1),n}$

1) $\underset{g}{Y^-}\ \underset{x}{U^+} \subseteq U^+ Y^- Y^+$

$x = vy =$ 

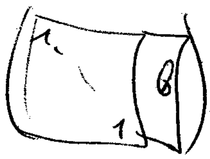
$gvy = \underset{U^+}{\underline{gvg^{-1}}}\, gy$

2) $StU(2n, R, \Delta) = \langle StUP_n, Y^- \rangle$

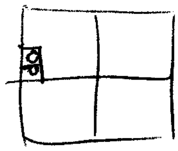
Усиление 1): $Y^- U^+ U^- \subseteq U^+ U^- Y^+ Y^-$

3) $sr R \leq n-2 \Rightarrow \forall v \in R^n$ – унимод.

$\exists\, b \in M(n-2, 2) : (e_{n-2}, b)\, v \in R^{n-2}$ – унимод.



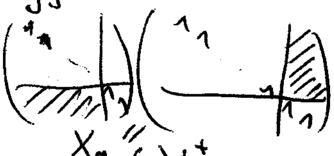
$Q := \{ g \in StUL_n \mid \pi(g)_{n-1,1} = \pi(g)_{n,1} = 0 \}$



4) $sr R \leq n-2 \Rightarrow \forall g \in StUL_n\ \exists\, x_g \in V^+,\ y_g \in V^-$

$y_g x_g g \in Q$

$y_g \in V^-$ $\quad v := \pi(g)_{*1} \in R^n$

 $v = \begin{pmatrix} \\ 0 \\ 0 \end{pmatrix} \}\, n-2$

$x_g \in V^+$

Следствие: $St\mathcal{U}\mathcal{U}_n^+\, St\mathcal{U}\mathcal{U}_n^-\, St\mathcal{U}L_n \subseteq \mathcal{U}^+\mathcal{U}^- Q$

$$St\mathcal{U}\mathcal{U}_n^+\, St\mathcal{U}\mathcal{U}_n^-\, g = \underbrace{St\mathcal{U}\mathcal{U}_n\, x^{-1}}_{\mathcal{U}^+}\Big(\underbrace{{}^{x_g}(St\mathcal{U}\mathcal{U}_n^-)\cdot y_g^{-1}}_{\mathcal{U}^-}\Big)\underbrace{(y_g x_g g)}_{Q}$$

Утверждение

$$\mathcal{Y}^-\, \mathcal{U}^+\mathcal{U}^- Q \subseteq \mathcal{U}^+\mathcal{U}^-\, St\mathcal{U}L_n\, St\mathcal{U}P_1$$

$a \in Q$ $\quad x \in a^{-1}\mathcal{Y}^- a \subseteq St\mathcal{U}\mathcal{U}_n^-$

$$\pi(ya)_{*1} = \pi(a)_{*1} \Rightarrow \pi(x) \in \begin{pmatrix} 1 & \\ 0 & \\ 0 \cdots & 1 \end{pmatrix}$$

$$\Rightarrow x \in \begin{pmatrix} & \\ & 1 \end{pmatrix} \subseteq St\mathcal{U}P_1$$

$$\mathcal{Y}^-\mathcal{U}^+\mathcal{U}^- a \subseteq \mathcal{U}^+\mathcal{U}^-\mathcal{Y}^+\mathcal{Y}^- a \subseteq \mathcal{U}^+\mathcal{U}^- a\,(\mathcal{Y}^+)^a(\mathcal{Y}^-)^a$$
$$\subseteq \mathcal{U}^+\mathcal{U}^- a\, St\mathcal{U}P_1$$

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$A := St\mathcal{U}P_n\, St\mathcal{U}\mathcal{U}_n^-\, St\mathcal{U}P_1$

Дост. д-ть, что $\mathcal{Y}^- A \subseteq A$

$$\mathcal{Y}^- A \subseteq \underbrace{St\mathcal{U}\mathcal{U}_n^+\, St\mathcal{U}\mathcal{U}_n^-\, St\mathcal{U}L_n}_{\subseteq\, \mathcal{U}^+\mathcal{U}^- Q}\, St\mathcal{U}P_1$$

$$\mathcal{Y}^-A \subseteq \mathcal{Y}^-\mathcal{U}^+\mathcal{U}^- Q\, St\mathcal{U}P_1 \subseteq A$$